8. Orthoscopy

The Background

From the first ideas to create progressive lenses (allowing to incorporate a cylindrical prescription) the inventors were well aware of the unavoidable peripheral astigmatism of optical surfaces incorporating a central meridian with changing power. At that time best-form single vision and bifocal lenses with minimized oblique astigmatism and oblique power error were in the center of concern of the ophthalmic optics engineers. The distortion of horizontal and vertical lines was a known phenomenon, but not considered as a main flaw in the optical quality of a lens.
The distortion of single vision lenses is rather small, but for progressive lenses with an add power until 3 D and more, the influence of distortion on the visual comfort is essential. Henry James Birchall attracts our intention in his patent US 2 475 275 to this fact and Bernard Maitenaz describes in patents US 3 687 528 and US 3 910 691 quantitatively the phenomenon of static and dynamic distortion. He describes several measures to construct a surface which maintains to the greatest possible extent the orientation of horizontal and vertical contours, i.e. a surface distinguished by a nearly orthoscopic image quality.

8.1 Distortion, definition and calculation model

In order to assess the distortion of a progressive lens we consider a rectangular grid in a vertical plane in a distance of e=1 m from the (vertical oriented) lens and calculate the displacements of these lines in the horizontal and vertical direction in the object plane, when we look through the progressive lens.

Horizontal Component Px of the Prismatic Effect

Fig 1 shows the horizontal plane z=const=z0 with the vertical projection of the coordinate axes x and y. The wearer looks at the point Obv of a vertical line intersecting this plane (angle of view σx). Because of the prismatic deflection δx he sees the object in X0, the x-cordinate difference to Obv (measured in cm) gives the horizontal prismatic power Px (in prism dioptres) of the progressive lens in the point (ξp, yp, z0). b' is the distance "center of rotation of the eye Z' to lens back vertex" and d is the center thickness of the lens (Fig 1 is a diagram for the general case z=const, b' and d are defined in the plane z=0 !).

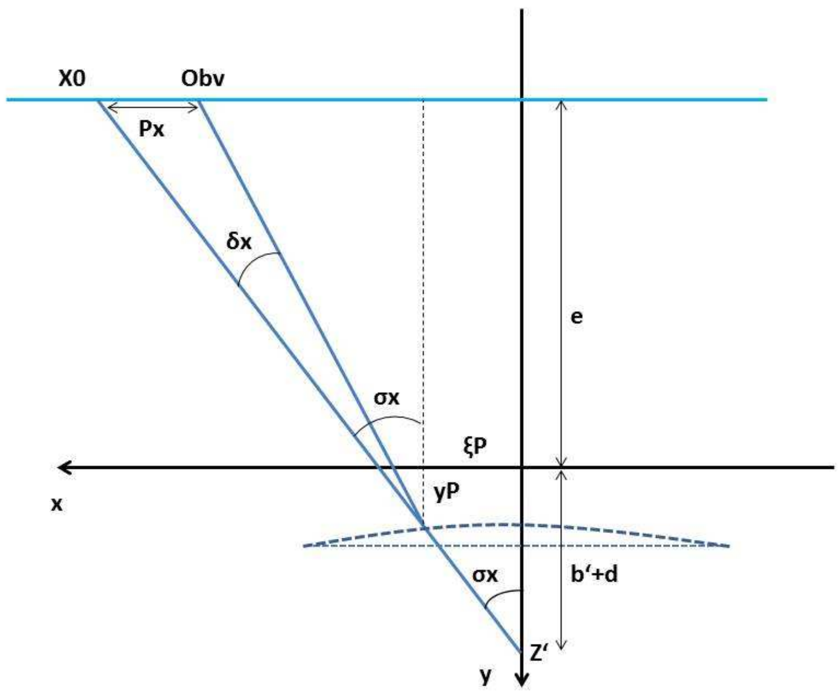

Fig 1

In order to calculate the distortion of the vertical lines we apply the following approximation:

*in a first step we determine the horizontal prism power Px assuming that the image in the object plane is the vertical X0=const (so necessarily with δx varying with z0 the object Obv would be a curved line).
* in a second step we assume that Obv are the equidistant vertical lines X0=const and that the deformation Px in a first approximation is the same as in step 1.

(It is possible to be more exact in calculating in the first step the average of the object position Obv and adding the deviation Px to this mean value, but in this case the vertical object lines are not equidistant and we have to interpolate.)

Vertical Component Pz of the Prismatic Effect

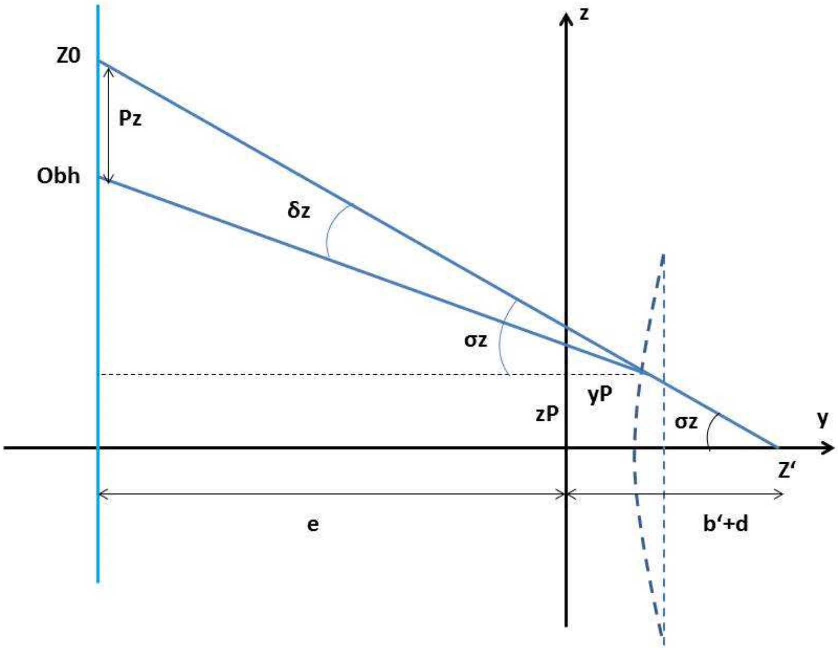

Fig 2

Fig 2 shows the plane $x = \xi$ =const= $\xi 0$ with the vertical projection of the the y- and z-coordinate axes . The designations follow the same logic as in Fig 1. Obh is a point of a horizontal line, for which we want to determine the deformation, Pz is the vertical prismatic power of the progressive lens in (ξ 0, yp, zp) .
For the calculation of the distortion of this horizontal line we proceed as before for the vertical:

*in a first step we determine the vertical prismatic deflection Pz assuming that the image in the object plane is the horizontal Z0=const (so necessarily with δz varying with $\xi 0$ the object Obh is a curved line).
* in a second step we assume Obh is situated on equidistant horizontal lines Z0=const and the deformation Pz in a first approximation is the same as in step1.

8.2 Non orthoscopic designs

We will look on the distortion properties of two designs, for which we have already analyzed their power and astigmatism patterns.
Fig 3 a and 3 b show the deformation of equidistant vertical and horizontal lines by a design following the "elephants trunk" concept (chapter 5). For the vertical lines we have chosen a 200 mm spacing, for the horizontal lines a spacing of 30 mm.

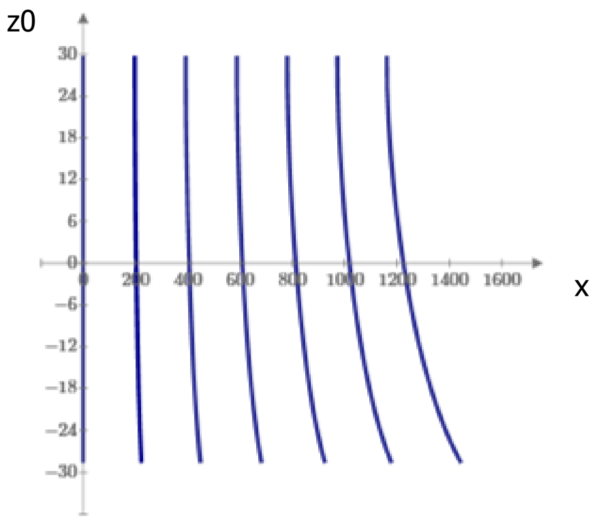

Fig 3 a

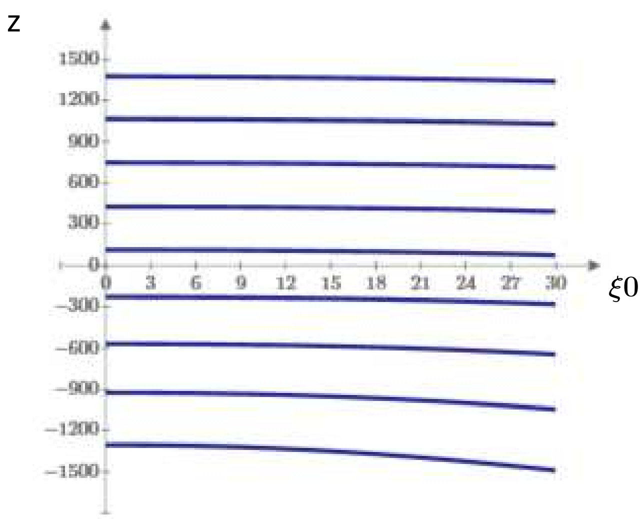

Fig 3 b

In Fig 3 a is x the coordinate of the distorted vertical in the grid plane, z0 the z-coordinate of the intersection point of the line of gaze with the progressive surface. In Fig 3 b is z the coordinate of the distorted horizontal in the grid plane, $\xi 0$ the x-coordinate of the intersection point of the line of gaze with the progressive surface (see Chapter 2).

We see a typical pincushion distortion as we know it from strong plus power single vision lenses. This is a consequence of the steady power increase along the main meridian and perpendicular to it, as it is illustrated by the graph in Fig 5 of chapter 5.

In the same way Fig 4 a and 4 b reflect the power profile characteristics of the Varilux 1 type design. We see the extended regions of stabilized far vision and near vision power with widely constant magnification, logically bigger in the NV than in the FV. Between these two zones there is a short, rather steep power increase with skew distortion. This kind of distortion is particularly difficult to tolerate under dynamic viewing conditions, i.e. in the conditions of a relative movement between object and progressive lens surface .

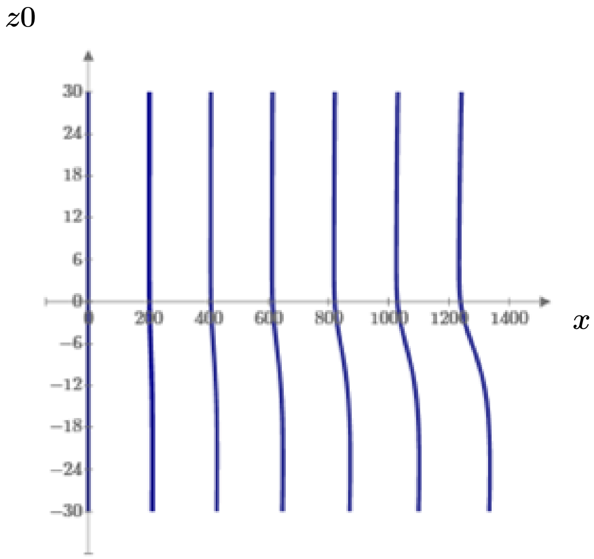

Fig 4 a

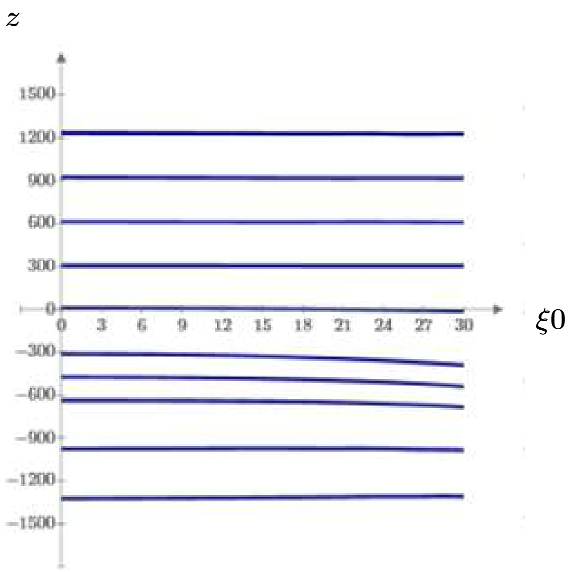

Fig 4 b

These experiences, expressed by the wearers during the continuous development of the Varilux concept in the 50's and 60's of the last century, gave Bernard Maitenaz the possibility to specify the conditions for a progressive surface with minimized distortion , for a design, which was almost orthoscopic. With the development of Varilux 2 Maitenaz left the concept of a bifocal-like progressive design. He discovered the importance of the lateral lens zones for the physiological aspects of the peripheral and dynamic vision.
Until that time the research of the optical engineers, conceiving single, bifocal and trifocal lenses, concentrated on visual acuity , a static vision characteristic determined by aberrations like astigmatism and oblique power error. But for the comfort of progressive lenses the distortion in the peripheral regions is as important as visual acuity in the central part. In the following approximate calculations we will see the big progress which the Varilux 2 surface represents compared to the first Varilux product. The reduction of astigmatism and its distribution on a larger area together with the introduction of secondary umbilical and isoprismatic lines are the essential building blocks of the "physiological progressive lens".
We will use the example from patent US 3 687 528 analyzed in Chapter 7.

8.3 Distortion of the Varilux 2 design

The calculation of the lens distortion in this chapter is an approximation. The exact determination calculates the ray path in 3 dimensions. As discussed above the considered model is a simplification, additionally the following considerations use small angle formulas.

8.3.1 The geometry and equations of the progressive surface

(all lengths are given in mm)

The equation of the principal meridian

$$Rm(z0) := 69.765 + 6.635 \cdot \sin\left(\frac{2\,\pi \cdot z0}{54.83} + 0.25\right)$$

Integrating the differential equation for the curvature K(z0) of the meridian

$$\frac{Fm''(z0)}{\left(1 + Fm'(z0)^2\right)^{\frac{3}{2}}} = K(z0) \qquad Fm(z0) = x1(z0)$$

$$x1'(z0) = x2(z0)$$

$$x2'(z0) = K(z0) \cdot \left(1 + x2(z0)^2\right)^{\frac{3}{2}}$$

$$K(z0) := \frac{1}{Rm(z0)}$$

$$D(z0, X) := \begin{bmatrix} X_1 \\ \left(1 + {X_1}^2\right)^{\frac{3}{2}} \cdot K(z0) \end{bmatrix}$$

$$X(z0) = \begin{bmatrix} x1(z0) \\ x2(z0) \end{bmatrix}$$

$u := 6.298$ mm $\qquad v := 0.448$

$$init := \begin{bmatrix} u \\ v \end{bmatrix}$$

$Z0i := 30$
$Z0f := -30$
$N := 100$

$$Sol := \mathrm{AdamsBDF}(init, Z0i, Z0f, N, D)$$

$$Sol = \begin{bmatrix} 30 & 6.298 & 0.448 \\ 29.4 & 6.033 & 0.436 \\ 28.8 & 5.775 & 0.425 \\ 28.2 & 5.523 & 0.413 \\ & & \vdots \end{bmatrix}$$

Calculating the main meridian as a function of z0: Fm(z0)

arranging the data in ascending order of z0

$$data := \mathrm{csort}(Sol, 0)$$

$$data = \begin{bmatrix} -30 & 7.253 & -0.513 \\ -29.4 & 6.949 & -0.501 \\ -28.8 & 6.652 & -0.489 \\ & & \vdots \end{bmatrix}$$

$Z0 := data^{\langle 0 \rangle}$ $X1 := data^{\langle 1 \rangle}$ $X2 := data^{\langle 2 \rangle}$

$S := \text{cspline}(Z0, X1)$

$Fm(z0) := \text{interp}(S, Z0, X1, z0)$

$S := \text{cspline}(Z0, X2)$

$D1Fm(z0) := \text{interp}(S, Z0, X2, z0)$

The characteristical data of the conic sections

The principal meridian is an umbilic. Thus the vertex equation of of a conic section in the (ξ,η)-system is

$$\eta = h \cdot \left(1 - \sqrt{1 - \frac{\xi^2}{Rm \cdot h}}\right)$$

where h is the half-axis of the conic section in direction of the η-axis. The calculations of h in chapter 7 define 3 zones

zone 1 for z0 values > -13.3: positive h-values, i.e. ellipses

zone 2 for z0-values between -13.3 and -18.6: negative h, hyperbolas

zone 3 for z0-values < -18.6: positive h, i.e. ellipses

for z0=-13.3 and z0=-18.6 there are two parabolas , i.e. h is becoming infinity

zone 1 : z0>-13.3

The resulting values for the parameter h and the coordinate z0 are given in the following matrix

$$data := \begin{bmatrix} -13.01 & 8000 \\ -12.52 & 4500 \\ -12.01 & 2650 \\ -11.02 & 1200 \\ -10.02 & 550 \\ -9.03 & 271 \\ -8.03 & 205 \\ -7.03 & 160 \\ & \vdots \end{bmatrix}$$

Calculation of the half-axis h1(z0) as a function of z0

$$Z0 := data^{\langle 0 \rangle} \qquad H := data^{\langle 1 \rangle}$$

$$S := \mathrm{cspline}(Z0, H)$$

$$h11(z0) := \mathrm{interp}(S, Z0, H, z0)$$

smoothing process

$$Z01 := \begin{bmatrix} -15 \\ -14.5 \\ -14 \\ -13.5 \\ -13 \\ -12.5 \\ -12 \\ -11.5 \\ \vdots \end{bmatrix} \qquad H1 := \begin{bmatrix} 5.92828 \cdot 10^{4} \\ 3.88307 \cdot 10^{4} \\ 2.41186 \cdot 10^{4} \\ 1.41442 \cdot 10^{4} \\ 7.90535 \cdot 10^{3} \\ \vdots \end{bmatrix}$$

$$ss1 := \mathrm{supsmooth}(Z01, H1)$$

$$S := \mathrm{cspline}(Z01, ss1)$$

$$h11(z0) := \mathrm{interp}(S, Z01, ss1, z0)$$

Taking into account the symmetry with respect to Vz=9° , i.e. z0=12.01 we obtain the half-diameter h1 for z0>-13.3 (infinity threshold , see below)

$$h1(z0) := \begin{cases} h11(z0) & \text{if } z0 \le 12.01 \\ h11(24.02 - z0) & \text{else} \end{cases}$$

zone 3: z0< -18.6: Calculation of the half-axis h3(z0) as a function of z0

The surface is symmetrical to the horizontal line Vz=-12° which corresponds to z0=-15.97 . So there is a second infinity point for -18.6 and the h-values for z0<-18.6 are given by

$$h3(z0) := h1(-2 \cdot 15.97 - z0)$$

zone 2: -18.6 < z0 < -13.3

The analysis of chapter 7 give the following h-values for z0 between -18.6 and -13.3

$$data := \begin{bmatrix} -18.42 & -7862 \\ -18.19 & -4142 \\ -17.93 & -2812 \\ -16.95 & -1347 \\ & \vdots \end{bmatrix}$$

Calculation of the half-axis h2(z0) as a function of z0

$Z0 := data^{\langle 0 \rangle}$ $\qquad$ $H := data^{\langle 1 \rangle}$

$$S := \mathrm{cspline}(Z0, H)$$

$$h21(z0) := \mathrm{interp}(S, Z0, H, z0)$$

$$h211(z0) := \frac{h21(z0) + h21(-2 \cdot 15.97 - z0)}{2}$$

Smoothing process

$$Z01 := \begin{bmatrix} -20.5 \\ -20.25 \\ -20 \\ -19.75 \\ -19.5 \\ -19.25 \\ -19 \\ -18.75 \\ -18.5 \\ \vdots \end{bmatrix} \qquad H1 := \begin{bmatrix} -(3.64785 \cdot 10^5) \\ -(2.71095 \cdot 10^5) \\ -(1.9525 \cdot 10^5) \\ -(1.35349 \cdot 10^5) \\ -(8.94921 \cdot 10^4) \\ -(5.57798 \cdot 10^4) \\ -(3.23116 \cdot 10^4) \\ \vdots \end{bmatrix}$$

$$ss2 := \mathrm{supsmooth}(Z01, H1)$$

$$S := \mathrm{cspline}(Z01, ss2)$$

$$h2(z0) := \mathrm{interp}(S, Z01, ss2, z0)$$

The function of the half-axis h(z0) covering the whole z0-range between -27 and +27 mm

$$h(z0) := \begin{array}{l} \text{if } z0 > -18.6 \\ \quad \begin{array}{l} \text{if } z0 > -13.3 \\ \quad h1(z0) \\ \text{else} \\ \quad h2(z0) \end{array} \\ \text{else} \\ \quad h3(z0) \end{array}$$

Orthogonal sections and derivatives

$$\eta(z0, \xi) := h(z0)\left(1 - \sqrt{1 - \frac{\xi^2}{h(z0) \cdot Rm(z0)}}\right)$$

$$x(z0, \xi) := \xi$$

$$y(z0, \xi) := Fm(z0) + \frac{\eta(z0, \xi)}{\sqrt{1 + D1Fm(z0)^2}}$$

$$z(z0, \xi) := z0 - \eta(z0, \xi) \cdot \frac{D1Fm(z0)}{\sqrt{1 + D1Fm(z0)^2}}$$

Definitions of the derivatives

$$r(z0,\xi) := \begin{bmatrix} x(z0,\xi) \\ y(z0,\xi) \\ z(z0,\xi) \end{bmatrix}$$

$$xz0(z0,\xi) := \frac{d}{dz0} x(z0,\xi)$$

$$yz0(z0,\xi) := \frac{d}{dz0} y(z0,\xi)$$

$$zz0(z0,\xi) := \frac{d}{dz0} z(z0,\xi)$$

$$rz0(z0,\xi) := \begin{bmatrix} xz0(z0,\xi) \\ yz0(z0,\xi) \\ zz0(z0,\xi) \end{bmatrix}$$

$$x\xi(z0,\xi) := \frac{d}{d\xi} x(z0,\xi)$$

$$y\xi(z0,\xi) := \frac{d}{d\xi} y(z0,\xi)$$

$$z\xi(z0,\xi) := \frac{d}{d\xi} z(z0,\xi)$$

$$r\xi(z0,\xi) := \begin{bmatrix} x\xi(z0,\xi) \\ y\xi(z0,\xi) \\ z\xi(z0,\xi) \end{bmatrix}$$

$$xz0z0(z0,\xi) := \frac{d^2}{dz0^2} x(z0,\xi)$$

$$yz0z0(z0,\xi) := \frac{d^2}{dz0^2} y(z0,\xi)$$

$$zz0z0(z0,\xi) := \frac{d^2}{dz0^2} z(z0,\xi)$$

$$rz0z0(z0,\xi) := \begin{bmatrix} xz0z0(z0,\xi) \\ yz0z0(z0,\xi) \\ zz0z0(z0,\xi) \end{bmatrix}$$

$$x\xi\xi(z0,\xi) := \frac{d^2}{d\xi^2} x(z0,\xi)$$

$$y\xi\xi(z0,\xi) := \frac{d^2}{d\xi^2} y(z0,\xi)$$

$$z\xi\xi(z0,\xi) := \frac{d^2}{d\xi^2} z(z0,\xi)$$

$$r\xi\xi(z0,\xi) := \begin{bmatrix} x\xi\xi(z0,\xi) \\ y\xi\xi(z0,\xi) \\ z\xi\xi(z0,\xi) \end{bmatrix}$$

$$xz0\xi(z0,\xi) := \frac{d}{d\xi} \left(\frac{d}{dz0} x(z0,\xi) \right)$$

$$yz0\xi(z0,\xi) := \frac{d}{d\xi} \left(\frac{d}{dz0} y(z0,\xi) \right)$$

$$zz0\xi(z0,\xi) := \frac{d}{d\xi} \left(\frac{d}{dz0} z(z0,\xi) \right)$$

$$rz0\xi(z0,\xi) := \begin{bmatrix} xz0\xi(z0,\xi) \\ yz0\xi(z0,\xi) \\ zz0\xi(z0,\xi) \end{bmatrix}$$

8.3.2 Distortion, quantitative results

Horizontal Component Px of the Prismatic Effect

Refering to Fig 1 we repeat the definitions

b': distance eye rotation center Z' to lens back vertex
d: lens center thickness
e: distance lens to object plane (grid)
(all distances in mm)

$$b' := 25.5 \qquad d := 3 \qquad e := 1000$$

We assume, that the lens should have zero power in the FV reference point at Vz=9° (see Fig 22 of the patent). Taking into account the power increase of 0.49 D between Vz=9° and Vz=0°, the value of the front surface radius of 71.29 mm at Vz=0° and applying the paraxial formula for the back vertex power S we obtain an approximate value for the back surface radius rb:

$$n := 1.525 \quad rf := 71.29 \qquad S := 0.49$$

$$rb := (n-1) \cdot \frac{\left(1 - \frac{d}{n} \cdot \frac{(n-1)}{rf}\right)}{\frac{(n-1)}{rf} - \frac{S}{1000} \cdot \left(1 - \frac{d}{n} \frac{(n-1)}{rf}\right)}$$

$$rb = 75.19$$

Additionally we define

$$A := e + b' + d \qquad B := b' + d$$

and using Fig 1 we calculate the the point of intersection (ξP,yP) of the line of view Z'X0 with the progressive surface using the following definitions.

$$C1(z0,X0):=\frac{\sqrt{1+D1Fm(z0)^{2}}}{h(z0)} \qquad C2(z0,X0):=\frac{X0^{2}}{A^{2}\cdot h(z0)\cdot Rm(z0)}$$

$$C(z0,X0):=C1(z0,X0)^{2}+C2(z0,X0)$$

$$D(z0,X0):=-2\cdot\left(Fm(z0)\cdot C1(z0,X0)^{2}+C1(z0,X0)+B\cdot C2(z0,X0)\right)$$

$$E(z0,X0):=Fm(z0)^{2}\cdot C1(z0,X0)^{2}+2\cdot Fm(z0)\cdot C1(z0,X0)+B^{2}\cdot C2(z0,X0)$$

Intersection point of the viewing line to X0 with the progressive surface (for z0=const)

$$yp(z0,X0):=\frac{-D(z0,X0)+\sqrt{D(z0,X0)^{2}-4\,C(z0,X0)\cdot E(z0,X0)}}{2\,C(z0,X0)}$$

$$yn(z0,X0):=\frac{-D(z0,X0)-\sqrt{D(z0,X0)^{2}-4\,C(z0,X0)\cdot E(z0,X0)}}{2\,C(z0,X0)}$$

For z0>-13.3 the orthogonal sections are ellipses , so we have to take the minus sign,i.e. yn, the same situation for z0<-18.6. Between-18.6 and -13.3 the section are hyperbolas , so the plus sign is correct.

```
yP(z0,X0) := || if z0 > -18.6
             ||   || if z0 > -13.34
             ||   ||   || yn(z0,X0)
             ||   || else
             ||   ||   || yp(z0,X0)
             || else
             ||   || yn(z0,X0)
```

The corresponding ξ-coordinate of the intersection point

$$\xi P(z0,X0):=\frac{X0\cdot(B-yP(z0,X0))}{A}$$

Now the intersection point with the sphere of the back surface with radius rb

$$F(X0):=1+\frac{X0^2}{A^2} \qquad G(X0):=-2\cdot\left(B\cdot\frac{X0^2}{A^2}+rb+d\right)$$

$$H(z0,X0):=B^2\cdot\frac{X0^2}{A^2}+2\cdot rb\cdot d+d^2+z0^2$$

$$yS(z0,X0):=\frac{-G(X0)-\sqrt{G(X0)^2-4\cdot F(X0)\cdot H(z0,X0)}}{2\cdot F(X0)}$$

$$\xi S(z0,X0):=\frac{X0\cdot(B-yS(z0,X0))}{A}$$

Calculation of the light ray deviation

αpx and αsx are the angles between the x-axis and the tangent of the progressive respectively spherical surface in the intersection points (ξP,yP) and (ξS,yS)

$$in1(z0,X0):=-yz0(z0,\xi P(z0,X0))\cdot z\xi(z0,\xi P(z0,X0))$$

$$in2(z0,X0):=y\xi(z0,\xi P(z0,X0))\cdot zz0(z0,\xi P(z0,X0))$$

$$\alpha px(z0,X0):=\operatorname{atan}\left(\frac{in1(z0,X0)+in2(z0,X0)}{zz0(z0,\xi P(z0,X0))}\right)$$

$$\alpha sx(z0,X0):=\operatorname{atan}\left(\frac{-\xi S(z0,X0)}{yS(z0,X0)-rb-d}\right)$$

Prism angle γx and horizontal prismatic deviation δx

$$\gamma x(z0,X0) := \alpha px(z0,X0) - \alpha sx(z0,X0)$$

$$\delta x(z0,X0) := (n-1)\cdot\gamma x(z0,X0)$$

$$\sigma(X0) := \operatorname{atan}\left(\frac{X0}{A}\right)$$

(1) prismatic deviation Px in the object plane e=1000 mm

$$Px(z0,X0) := X0 - (e + yP(z0,X0))\cdot\tan(\sigma(X0) - \delta x(z0,X0)) - \xi P(z0,X0)$$

(2) now we assume that the object is at X0=const and the prismatic power is in a first approximation identical to Px (z0,X0). Thus we get for the distorted vertical line DVx(z0,X0)

$$DVx(z0,X0) := X0 + Px(z0,X0)$$

If we plot the function DVx(z0,X0) as x-coordinate of the distorted vertical in the grid plane for the parameter values X0=0, 200, 400....1000 we get Fig 5.

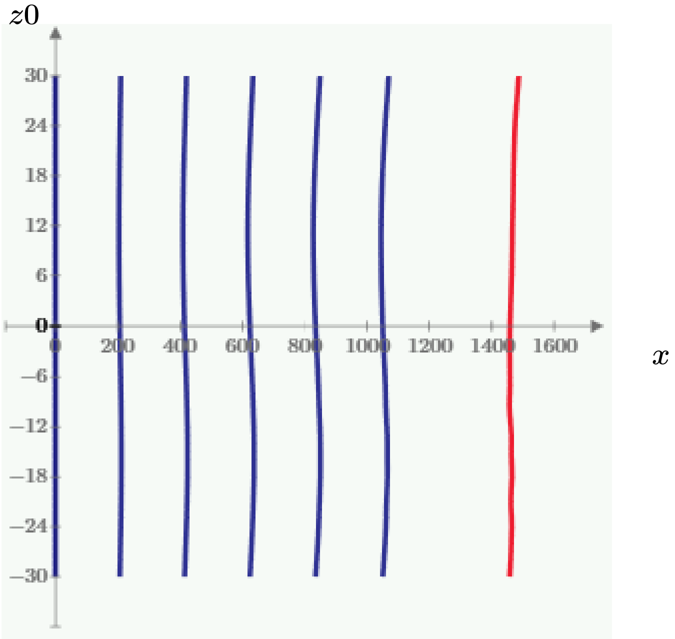

Fig 5

The vertical lines show the expected general characteristics. Descending from the far vision to the near vision part the higher magnification of the add power bulges the lines slightly to higher x-values. The increasing power of the sine -like power profile of the main meridian on the top of the lens and the power decrease at the bottom cause the somewhat wavy structure, which also shows up in Fig 21 of patent US 3 687 528. The deformation of the vertical lines in the periphery is neatly smaller than for Varilux 1. By trial and error you find a substantially straight line at X0=1370. Calculating the corresponding point on the progressive surface we get for the spherical coordinate Vx the value 22.5° , which is, according to the patent, the position of the vertical isoprismatic line.

$z0 := 0 \quad X0 := 1370$

$$\xi P(z0, X0) := \frac{X0 \cdot (B - yP(z0, X0))}{A}$$

$$Vx(z0, \xi) := \frac{180}{\pi} \cdot \operatorname{asin}\left(\frac{x(z0, \xi)}{76.80}\right)$$

$$\xi P(z0, X0) = 29.4$$

$$Vx(0, 29.42) = 22.5$$

Vertical Component Pz of the Prismatic Effect

For the calculation of the distortion of a horizontal line we proceed as before for the vertical, using the notations of Fig 2 showing the ray path in the plane x=ξ=const=ξ0.

At first we have to determine the point of intersection (yP,zP) between the line of view Z'Z0 and the progressive surface. In order to describe yP and zP by the given parameters Z0 and $\xi 0$ we have to determine the value z0 for the point on the principal meridian, which belongs to the orthogonal section passing through the point of intersection (yP,zP).
We use the root-function and its formalism of the *Mathcad* software and introduce a new auxiliary coordinate ξa:

$$\xi 0(\xi a) := \xi a - 1$$

$$f2(z0, \xi a) := y(z0, \xi 0(\xi a))$$

$$f1(z0, \xi a, Z0) := B - \frac{A \cdot z(z0, \xi 0(\xi a))}{Z0}$$

$$S(\xi a, z0, Z0) := \mathbf{root}(f1(z0, \xi a, Z0) - f2(z0, \xi a), z0)$$

$Z0 := 1$ (necessary to put a Z0 value because this page is part of the *Mathcad* program)

With

$\xi a := 1, 2 .. 31$ $\quad z0_0 := 1$

corresponding to the $\xi 0$ range from 0 to 30 mm and an initial estimation value , we get for the z0-value of the orthogonal section passing through point (yP,zP):

$$z0_{\xi a} := S\left(\xi a, z0_{\xi a - 1}, Z0\right)$$

and for the coordinates yP and zP of the intersection point on the progressive surface

$$yP(Z0,\xi a):=B-\frac{A\cdot z\left(S\left(\xi a,z0_{\xi a-1},Z0\right),\xi0(\xi a)\right)}{Z0}$$

$$zP(Z0,\xi a):=z\left(S\left(\xi a,z0_{\xi a-1},Z0\right),\xi0(\xi a)\right)$$

For the point of intersection between the viewing line Z'Z0 and the sphere of the back surface we obtain

$$F(Z0):=1+\frac{Z0^2}{A^2}\qquad G(Z0):=-2\cdot\left(B\cdot\frac{Z0^2}{A^2}+rb+d\right)$$

$$H(Z0,\xi a):=B^2\cdot\frac{Z0^2}{A^2}+2\cdot rb\cdot d+d^2+\xi0(\xi a)^2$$

$$yS(Z0,\xi a):=\frac{-G(Z0)-\sqrt{G(Z0)^2-4\cdot F(Z0)\cdot H(Z0,\xi0(\xi a))}}{2\cdot F(Z0)}$$

$$zS(Z0,\xi a):=\frac{Z0\cdot(B-yS(Z0,\xi0(\xi a)))}{A}$$

So for the deflection angles αpz and αsz holds

$$\alpha pz(Z0,\xi a):=\operatorname{atan}\left(\frac{yz0\left(S\left(\xi a,z0_{\xi a-1},Z0\right),\xi0(\xi a)\right)}{zz0\left(S\left(\xi a,z0_{\xi a-1},Z0\right),\xi0(\xi a)\right)}\right)$$

$$\alpha sz(Z0,\xi a):=\operatorname{atan}\left(\frac{-zS(Z0,\xi0(\xi a))}{(yS(Z0,\xi0(\xi a))-rb-d)}\right)$$

and the prism angle γz and the vertical prismatic deviation angle δz are

$$\gamma z(Z0,\xi a) := \alpha pz(Z0,\xi a) - \alpha sz(Z0,\xi a)$$

$$\delta z(Z0,\xi a) := (n-1)\cdot\gamma z(Z0,\xi a)$$

$$\sigma(Z0) := \mathrm{atan}\left(\frac{Z0}{A}\right)$$

(1) prismatic power Pz in the object plane e=1000 mm

$$Pz(Z0,\xi a) := Z0 - z\left(S\left(\xi a, z0_{\xi a-1}, Z0\right), \xi 0(\xi a)\right) - (e + yP(Z0,\xi a))\cdot\tan(\sigma(Z0) - \delta z(Z0,\xi a))$$

(2) now we assume that the object is at Z0=const and the prismatic deviation is in a first approximation identical to Pz (Z0,ξa) and we get for the distorted horizontal line DHz(Z0,ξa)

$$DHz(Z0,\xi a) := Z0 + Pz(Z0,\xi a)$$

If we plot the function DHz(Z0,ξa) as z-coordinate of the distorted horizontal in the grid plane for the parameter values Z0=-1200, -900, -600....1200 we get Fig 6.

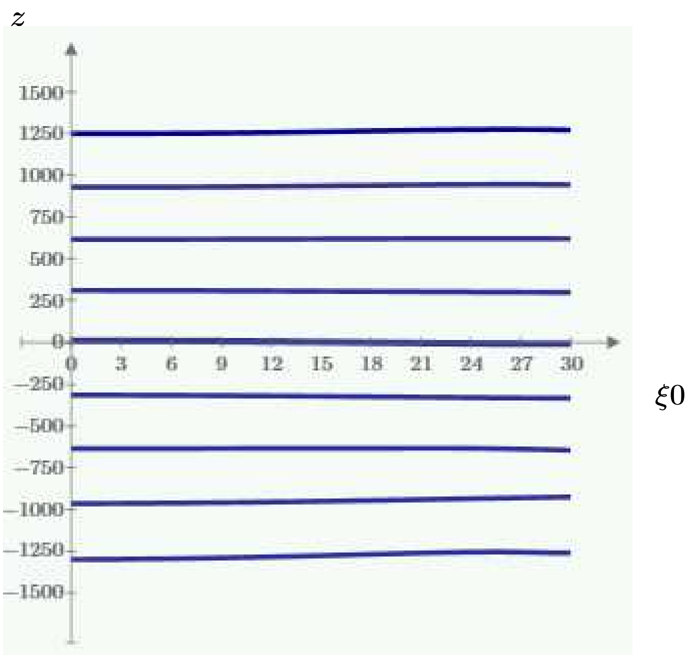

Fig 6

The distortion of the horizontals in the lateral progression zone of the Varilux 2 design is relatively weak compared to the deformation observed for Varilux 1.
In order to analyze the characteristics and differences more clearly, you can consider particularly the value of the prismatic deflection Pz. By this way in the FV-part and in the NV-region two horizontals can be identified, where the vertical prismatic component is roughly constant.
So the Varilux 2 design with its horizontal umbilical lines and vertical isoprismatic lines reduces distinctly the prismatic deformation of the horizontal and vertical contours of visual objects . As these contours are essential for our orientation in daily life the development of Varilux 2 was an outstanding progress for the visual comfort of the progressive lens wearer.